Archives LD 175 .A40K The 579

# DOMESTIC ENERGY POLICY AND THE CRUDE

# A THESIS PRESENTED to the FACULTY of the GRADUATE SCHOOL APPALACHIAN STATE UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE MASTER OF BUSINESS ADMINISTRATION

> BY DAVID WILLIAM BROOKS MAY 1981

35-1 .31

# DOMESTIC ENERGY POLICY AND THE CRUDE OIL PRODUCTION DECLINE CURVE

Ву

#### DAVID W. BROOKS

Approved By

5

Associate Professor of Economics Committee Chairman

m nº4

Associate Professor of Finance

Associate Professor of Management

Joyce VLaure

•

Dean of the Graduate School

#### ABSTRACT

The United States, reacting to rising prices and supply uncertainties of imported energy, has begun to move aggressively to develop its untapped domestic energy resources. Prior to the Arab oil embargo that began in October of 1973, the general feeling was that the U.S. oil resources were almost limitless. The experiences of the last eight years have taught us differently.

There have been a large number of analytical estimates of U.S. oil supply potential. Experts do not agree on the appropriate method for forecasting the availability of a nonrenewable resource. The M. K. Hubbert model has made extremely accurate predictions in the past but, its accuracy is questionable in the future due to the instability in the petroleum industry.

This paper builds a model to predict future supplies of oil. The relevant variables in the model are the crude oil production decline rate, the initial price per barrel of crude oil, the rate of change in price, the elasticity of supply with respect to price, and the time period in question,

It is proven that production of oil from an individual well decreases through time because of the decline rate. The decline in production due to the decline rate can be offset partially or completely by an increase in the price of oil. Also, it is proven that a decrease in the price of oil will cause a decrease in production in excess of the decline rate.

It is necessary to accurately predict future supplies of oil before it is possible to build an effective domestic energy policy. In order to project future supplies of oil this paper proves that it is necessary to include both geologic and economic considerations.

## TABLE OF CONTENTS

2

Chapter 1:	Domestic Energy Policy and Production Decline Rates	1-11
Chapter 2:	Review of Empirical Literature and the Importance of the Decline Rate in Forecasting Crude Oil Supplies	1-11
Chapter 3:	The Derivation of a Production Decline Model	1-13
Chapter 4:	Empirical Estimations and Simulations	1-10
Chapter 5:	Conclusions and Policy Implications	1 - 4
Appendix I		
Appendix II		
Appendix III		
Appendix IV		

Appendix V

## LIST OF FIGURES

P	а	n	0	#
	u	У	C	11

	Figure 1.	The Production Decline Curve
	Figure 2.	The Marginal Cost Curve for an Individual Oil Well
	Figure 3.	Production Decline Over Time With No Real Change in Price
	Figure 4.	The Crude Oil Production Decline Curve Given a Constant Decline Rate and No Change in Price
	Figure 5.	Relationship of Quantity to Changes in the Price
	Figure 6.	Production Changes Over Time Given a Decline Rate, Rate of Change in Price, and Elasticity of Supply 9, Ch. 3
	Figure 7.	The Crude Oil Production Decline Curve Given the Quantities in Figure Six10, Ch. 3
	Figure 8.	The Offset in the Decline Rate Due to an Increase in Price
	Figure 9.	Change in Quantity Due to Decline Rate is Exactly Offset By the Change in Price
1	Figure 10.	Change in Quantity Due to Decline Rate is More than Offset By Change in Price
1	Figure 11.	Effect on Output of Decrease in Price Appendix V
1	Figure 12.	Decline Curve From Figure 11 for Decrease in Price Appendix V

#### CHAPTER ONE

## DOMESTIC ENERGY POLICY AND PRODUCTION DECLINE RATES

The United States, reacting to rising prices and supply uncertainties of imported energy, has begun to move aggressively to develop its untapped domestic energy resources. Prior to the Arab oil embargo that began in October 1973, the general feeling was that the U.S. oil resources were almost limitless. The experiences of the last eight years have taught us differently.

In the early 1960's the United States was very secure with its oil production. Consumption of oil was increasing by small amounts each year. Increases in U.S. production rates were ample to meet the increased demands. Demand and supply of oil were stable.

During this period of time the economic structure of the country was also changing. Overall output in the economy was increasing. In order to increase output the business sector had to employ more resources. Consistent with profit maximization techniques they employed those additional resources which cost least. Through the years (from 1958 to 1972) energy, in the form of oil, had been decreasing in cost relative to the other factors of production. For many years the U.S. had enjoyed abundant low cost supplies of domestic energy.<sup>1</sup> Instead of increasing output through

<sup>&</sup>lt;sup>1</sup>Grayson, Lesiie E. Editor. <u>Economics of Energy</u> (Princeton, N.J., The Darwin Press) 1975, p. 97.

increases in inputs such as labor, the U.S. economy was increasing energy inputs such as oil. Oil became a larger part of the price of products than it had been in the past.

Along with additional real output comes additional real income and increased prosperity. This prosperity was most easily noted through the increase in the number of individuals who could afford an automobile and, also, through those buying a second and even third car within one family. With more automobiles being driven everyday, more and more gasoline was demanded. In order to have gasoline one must first have crude oil. The U.S. was nearing the time when domestic crude oil production would no longer be adequate to keep up with continually rising consumption. Although the rate of increase in demand was expected to slow, domestic production of petroleum could not be expected to ever again be able to meet demand.<sup>2</sup> At the time the answer appeared simple on the surface -- import more oil from foreign sources. Foreign oil wasn't particularly expensive. There was plenty of it and the imports would cause no immediate strain on the trade balance.

<sup>&</sup>lt;sup>2</sup>Cook, Earl. <u>Forecasting Depletion</u> In Background Readings on Energy Policy. Edited by the Committee on Ways and Means. (Washington D.C.: U.S. Government Printing Office) 1979, p. 96.

Thus the U.S. dependence upon foreign oil imports had begun and the stage was set for the 1973 Arab Oil Embargo. By September of 1973 we were importing about 38% of our consumption of all petroleum products.<sup>3</sup>

The dangers of increased foreign oil imports into this country and thus increased dependence upon foreign oil had been considered in the past. The limiting of oil imports began in the late 1950's and was rationalized on the basis of national security. On March 10, 1959 a Presidential Proclamation imposed mandatory controls on the imports of oil. The proclamation stated security as its principle ground and economic protection as its second point. It also stated that "Energy is not solely a matter of economics, it is a matter also, of security policy."<sup>4</sup>

Through the 1960's and very early 1970's (1970,'71, '72) the relative price of oil had fallen in comparison to other goods. In 1971 the Teheran-Tripoli was concluded. At this conference two major occurrences took place. The first was an agreement among middle east nations to halt the secular decline in world crude oil prices which had

<sup>3</sup>Tietenberg, Thomas H. <u>Energy Policy and Planning</u>. (D.C. Heath and Co., Lexington, Massachusetts. Lexington Books) 1976, p. 21.

<sup>4</sup>McKie, James W. "United States and Canadian Energy Policy" <u>Oil in the Seventies</u>. The Fraser Institute, 1977, p. 247.

begun in 1956. Secondly, and the most important, the Organization of Petroleum Exporting Countries, OPEC, had become relatively united and established itself as a force to be reckoned with by the rest of the world, particularly the western world and Europe.<sup>5</sup>

On October 26, 1973 the Persian Gulf members of OPEC decided to raise prices unilaterally by 70%. On the following day OPEC as a whole accepted this policy.<sup>6</sup> This action caused the price of crude oil to quadruple over the next five years.

The quadrupling of crude oil prices caused a great deal of trouble throughout the free world. These troubles are entailed in three main problems: 1) The import vulnerability problem, 2) The balance of payments problem, and 3) The foreign policy problem. The simple fact that the U.S. economy was dependent upon imported foreign oil for a large part of its energy needs made this economy very susceptible to potentially large externally generated shocks.<sup>7</sup>

<sup>5</sup>Tietenberg, Thomas H. p. 26. <sup>6</sup>Ibid, p. 36. <sup>7</sup>Ibid. p. 36.

This effectively reduces the dependability of economic plans and forecasts. The U.S. economy had one distinct vulnerability or Achille's heel; imported oil. The balance of payments problem is the most fundamental and wide-ranging problem of the three. The consuming nations (those nations importing OPEC oil) were putting much larger amounts of money into imports for foreign oil than were being met by increases in exports -- again weakening the U.S. economy.<sup>8</sup> Neil H. Jacoby states the problem like this, "The quadrupling of the price of oil fundamentally changed the international balances of payments of virtually every non-Communist country and brought about a revision of national economic plans. At some point the OPEC oil prices could precipitate a breakdown of the international financial system."<sup>9</sup>

The impact on American foreign policy is highly related to the first two problems. The dependence upon OPEC for a critical input made the U.S. vulnerable on several policy fronts. First there was the threat of another embargo, which was now a credible possibility, and raised the

<sup>8</sup>Tietenberg, Thomas H. p. 36.

<sup>9</sup>Jacoby, Neil H. "Multinational Oil" <u>A Study in Industrial</u> <u>Dynamics</u>. (New York: McMillan Publishing Co., Inc.) 1974, pp. 263, 267.

idea that this could be used to gain favorable recognition of the Arab-Israeli dispute. Second, the large dollar flows to the Arab nations gave them an enormous financial resource base. This could be used to gain political leverage in several different ways. The money could be spent on arms or even the technology to produce nuclear weapons. Also, the money could be used to buy major international corporations to possibly gain control of other markets as well as petroleum. The strategic placement of funds in short term financial assets could be used as a basis for blackmail due to their importance on the world financial markets. Sudden shifts of large quantities of short-term deposits could put the affected banks in a "precarious financial position."<sup>10</sup>

In short, the effectiveness of American foreign policy was in question. The United States' ability to coax international forces in ways congenial to the long-term interests of the U.S. was, and is, in jeopardy.<sup>11</sup>

An industrialized nation must have a continuing inflow of mineral and energy resources to maintain its standard of living and national strength.<sup>12</sup> In 1973 the seed was planted through the Arab Oil Embargo. Now the potential has been born to render U.S. foreign policy useless and ineffective

<sup>10</sup>Tietenberg, Thomas H. p. 37.
<sup>11</sup>Ibid., p. 38.
<sup>12</sup>Cook, Earl, p. 96.

as a means of protecting the American way of life. Whether this could actually occur or not is irrelevant. The potential still remains, and this potential is centered around, and inherent within, the production of crude oil. The best way for this country to improve its foreign policy in view of the above weaknesses is to improve its domestic energy policy.

This illustrates the importance of trying to forecast the availability and costs of the vital industrial resources, In order to draft a viable domestic energy mainly oil. policy we must be able to forecast the supply of oil. In order to forecast the supply of oil we must take into consideration the production decline rate for an oil well. The production decline rate is the rate at which production of oil from a well decreases through time because of decreased pressure within the well. The decline curve for U.S. crude oil production for the period 1970-2000 is exhibited in Figure One. This curve illustrates the constant decrease in the production of oil for this thirty-year period. Because both availability and cost, in the case of non-renewable resources, depend significantly on depletion rates (or as termed in this discussion, decline rates) and on the control of those resources, national materials strategies, and thus domestic energy policy, need to be based solidly



FIGURE ONE: The Production Decline Curve (historically projected) from 1970 to 2000.<sup>1</sup> This represents aggregate U.S. oil supply.

<sup>1</sup>Enhanced Oil Recovery, "An Analysis of the Potential for Enhanced Oil Recovery from Known Fields in the United States -- 1976 to 2000," <u>The National Petroleum Council</u>. December 1976, p. 216. on forecasts of depletion rates for both domestic and foreign reserves (in the current text we deal with only domestic reserves).<sup>13</sup>

There is pressure in an untapped oil well. This pressure is present for two main reasons. The first and most basic reason is because the oil has been formed in a finite geologic area. As more oil formed it became more tightly packed. The second reason is because of the gases in and around the oil. Once tapped, the oil from a well flows out freely because of the pressure within the well.<sup>14</sup> "Removal of oil leads necessarily to a reduction in pressure and a redistribution of the reservoir fluids; in consequence, the practicable rate of production declines with time."<sup>15</sup>

Every oil well has a life determined by 1) the amount of oil that can be extracted from it at a profit and 2) the rate of extraction.<sup>16</sup> Given the same inputs and costs of production for oil, the output will decline over time and the cost per barrel will increase. Because oil will in

<sup>13</sup>Ibid., p. 96.

<sup>16</sup>Cock, Earl, p. 90.

<sup>&</sup>lt;sup>14</sup>How freely the oil flows depends upon many variables, all of which are unimportant in the progression of the current topic matter.

<sup>&</sup>lt;sup>15</sup>May, C.J. "The Production of Oil and Gas" G.D. Hobson, <u>Modern Petroleum Technology</u>. (New York: John Wiley and Sons) 1975, p. 145.

all probability remain our main energy source for the next twenty years it is of great importance to try to increase the amount of oil that can be extracted at a profit. The more domestic oil the U.S. has, the more stable is its industrial base, standard of living, and industrial strength. The actual physical decline rate cannot be changed but, it can be offset or added to by changes in prices, technological advancements, changes in taxes, regulatory restrictions, expectations, and enhanced oil recovery techniques. "Enhanced oil recovery techniques are known techniques for recovering additional oil from a petroleum reservoir beyond that economically recoverable by conventional primary and secondary recovery methods." Three of the most common enhanced oil recovery techniques are the thermal recovery process, the miscible flooding process, and the chemical flooding process.<sup>17</sup> All of these methods involve injecting something into an oil well in order to increase the flow of oil from the well.

Given that we know or can estimate the demand for oil over the next twenty years, it is important to forecast the supply of oil so that we can develop a timely, accurate and effective domestic energy policy. The decline rate is an

<sup>&</sup>lt;sup>17</sup>Enhanced Oil Recovery, p. 218. For a detailed discussion of enhanced oil recovery techniques see this reference.

essential variable in forecasting the supply of oil and therefore has important policy implications.

The second chapter will discuss crude oil forecasting and will concentrate on the M.K. Hubbert model and the importance of the decline rate in policy formation. This chapter will also contain an explanation of the variables to be used to build a model in the third chapter. The third chapter is concerned only with developing a mathematical model of the crude oil production decline curve. Once this model is derived it will be demonstrated, in chapter four, how the decline curve shifts for different values of the variables in the model. Finally, in chapter five, conclusions will be drawn about how the variables in the model are interrelated and the policy implications of these relationships.

There are several limiting factors to this study. These are factors that are held constant in order to determine how a change in price will ultimately affect oil supply, <u>ceteris</u> <u>paribus</u>. A more general model than is built in this paper might include the following points.

 The decline rate will not actually remain a constant percentage. It will decrease over time. The largest decline in output will be in the early stages of extraction. After this, the percentage decrease in production will drop.

2) The rate of change in price will not be constant.

- 3) The decline rate can be influenced by the spacing of wells in the oil field, the gas concentration in the well, and the size of the hole drilled into the well.
- 4) Technology is held constant because there is no means to quantitatively account for changes in technology. Technology can increase production if it results in lower costs, increased demand, or more efficient utilization; it can decrease production if it favors a competing resource.<sup>18</sup>
- 5) The elasticity of supply with respect to price will not actually remain constant over time.
- 6) This study does not include any effects upon the supply of oil due to the windfall profits tax.<sup>19</sup>
- The effects of enhanced oil recovery are not discussed in depth.
- There is no attempt to combine individual forecasts in an effort to produce an aggregate forecast.
- 9) This study holds constant human activities, such as wars and depressions, because of their unpredictability and the drastic effects these events have on petroleum production.

<sup>18</sup>Cook, Earl, p. 97.

<sup>19</sup> For an analysis on the effects of the windfall profits tax see Phillip E. Verleger, Jr., "An Assessment of the Windfall Profits Tax on Crude Oil Supply" <u>The Energy</u> <u>Journal</u>, Vol. 1, Number 4, October, 1980. p. 41-58. Also see Steven W. Millsaps and Mack Ott "The Effects of the Windfall Profits Tax on Domestic Petroleum Exploration, Development, and Production." Paper prepared for L.A.S.E.R. Conference on Federal Land Policies. November, 1980.

#### CHAPTER TWO

# REVIEW OF EMPIRICAL LITERATURE AND THE IMPORTANCE OF THE DECLINE RATE IN FORECASTING CRUDE OIL SUPPLIES

There have been a large number of analytical estimates of U.S. oil and gas supply potential.<sup>1</sup> They are notable for their diversity of approaches and the wide difference in their answers.<sup>2</sup> Experts do not agree on the method appropriate for forecasting availability of a non-renewable resource. Some maintain that demand determines availability. Others hold that geologic criteria determine availability, but these latter disagree strongly on the appropriate method of forecasting and in the assumptions which are proper or needed.<sup>3</sup>

Any long-range forecast of oil supply contains implicit assumptions about the advancement of technology. Because technology is quantitatively unpredictable, the reliability of long-range forecasts is questionable. However, depletion forecasts of short and intermediate range for specific resources in limited geographic areas are extremely useful in allocating investments and adjusting political strategies and policies.<sup>4</sup> The following are the most common methods of forecasting crude oil supply.

<sup>2</sup>Erickson, Ed; Steven W. Millsaps, William L. Peters, "The Benefits of Phased Decontrol of the Price of All U.S. Crude Oil Production." A paper prepared for the <u>American Petroleum Institute</u>. March 28, 1979, p. 6.

<sup>3</sup>Cook, Earl, p. 134.

<sup>4</sup>I bid., p. 135.

<sup>&</sup>lt;sup>1</sup>Throughout the rest of this paper the terms "crude oil, oil, and petroleum will be used interchangeably and considered to be synonomous.

Volumetric analysis of supply is a method based upon the fact that petroleum reserves are only found in sedimentary rock formations.<sup>5</sup> This method assumes that one can determine or measure the reserves per unit volume of the sediments in the region. With a particular set of economic conditions one should be able to apply the same recovery fraction of reserves per unit volume of sediments to other partially developed or undeveloped regions and thus determine the remaining recoverable reserves in that region. In an extension to this approach the original recovery fraction obtained depends very heavily upon the economic conditions (i.e. price) at the time. This, and other problems, cause the volumetric approach or extensions of it to be used only in obtaining very rough approximations of the supply of oil.<sup>6</sup>

Another geologic type analysis of supply is called the prospect potential equation. This analysis is postulated:

Prospect potential = Volume of Pores in Trap X Hydrocarbon fraction X Recovery X Engineering factors The right side of this equation states quantities that affect the ultimate size of the oil field prospect.

<sup>5</sup>Uhler, Russel, p. 20. <sup>6</sup>Ibid., p. 21.

These quantities are subject to uncertainty and are given subjective estimates of their possibilities. This method has considerable appeal because it uses the best current information available and is very flexible. However, it has one significant technical problem. This problem is termed multiple colinearity which means the results are subject to error because of correlations between the independent variables.<sup>7</sup>

The M. King Hubbert model must be discussed because of its substantially lower estimate of the undiscovered U.S. oil resource base than that derived from alternative approaches and because in at least one instance it has demonstrated remarkable predictive power.<sup>8</sup>

Hubbert's approach is in essence a categorical rejection of the volumetric analysis and other geologic methods for estimating supply potential. He makes these rejections because their estimates are not based upon actual drilling results.<sup>9</sup> M.K. Hubbert (1962) states "the only possible way we have of determining how much oil the United States will produce is by pure empiricism, based on our actual experience in exploration and production of petroleum."

<sup>7</sup>Uhler, Russell, p. 23.

<sup>8</sup>Erickson, Millsaps, Peters, p. 6.

<sup>9</sup>Uhler, Russell, p. 26.

Hubbert combines all the complexities of the U.S. petroleum industry into three time series: 1) the rate of annual crude oil discoveries; 2) the rate of annual crude oil production; and 3) the rate of annual change of crude oil reserves. These time series are arranged in a model to predict the future based solely on the past, or pre-1962 economic and technological trends and relationships. In 1961 Hubbert predicted, with this model, that U.S. oil production (lower 48 states and all adjoining offshore areas) would reach a maximum peak in 1969 and then decline. U.S. oil production actually peaked in 1970 and then began to decline along a path generally consistent with Hubbert's prediction.<sup>10</sup>

J.T. Ryan also has criticized the model (1974) on the grounds that Hubbert's production curve, a normal bell shaped curve, was chosen on the purely empirical ground that it provided a good fit to past data. He contends this doesn't necessarily mean the analytical function will continue to fit in the future. The only other distinct criticism is that Hubbert's model fits well in the aggregate but not for an

<sup>10</sup>Erickson, Millsaps, Peters pages 6 and 7.

For a definitive detailed analysis and critique of the Hubbert approach, see DeVerle P. Harris, "Conventional Crude Oil Resources of the United States: Recent Estimates, Methods for Estimation and Policy Considerations," <u>Materials and Society</u>, Vol. 1, 1977, pp. 263-286.

oil field or single well.

Considering the economic changes of the last decade and the tremendous leaps in the price of crude oil since 1973, any model that does not allow for changes in economic conditions is not adequate in forecasting the supply of oil. For every valuable mineral deposit there is an intrinsic limit of exploitation determined by its geologic boundaries. There is also an economic limit of exploitation determined by the value of the product at the mine mouth or wellhead, and the rate and cost of extraction. Although the intrinsic limit does not change with time, the economic limit While this economic limit cannot physically change the may. geologic boundaries it can, to a degree, render those boundaries meaningless in the determination of whether or not to market the product. As long as a profit can be made, and the mineral is geologically available, it will be ex-When a mineral can no longer be mined profitably tracted. the geologic boundaries are again of little importance.<sup>11</sup>

The most applicable model, then, would be one that combines all of the methods previously mentioned and, also allows for changes in economic conditions. In fact it is

<sup>11</sup>Cook, Earl, p. 100.

imperative that changes in the price of oil and the natural decline in production through time be taken into consideration when forecasting oil supply.<sup>12</sup> It would also be better to have a model that fits for an individual oil well and then combine the individual results to produce an aggregate forecast (however, this paper deals only with building a model to represent a single oil well).

Demand and supply projections should actually be carried out simultaneously, not separately. To regard demand for any commodity as independent of the supply seems hazardous to planning. This is especially true for non-renewable resources where supplies may be limited by physical factors and susceptible to constraint by political action.<sup>13</sup> Because of no attempt here to forecast demand this study will not be specifically to forecast aggregate supply but rather to model the decline curve. However, in order to model the decline curve it is necessary to forecast supply for an individual oil well. A general formula will be derived that will give the output of oil in any time period in the future. The variables necessary to predict the supply of oil from an individual well have now been determined. These variables

<sup>12</sup>Uhler, Russell, p. 11.

<sup>13</sup>Cook, Earl p. 102.

are: 1) the production decline rate; 2) the price of oil;3) the rate of change in the price of oil; and 4) the elasticity of supply with respect to price.

The production decline rate is the geologic part of the model. The production of oil from a well will decline through time at the rate equal to the decline rate due to decreased pressure in the well, given the other variables remain constant (as exhibited in Figure Three, p. 3, Ch. 3) Because we need to optimize and in some cases maximize the production of oil from a well we attempt to offset the decline rate by changes in the other variables. The decline rate is increased by lower oil prices and decreased by higher The decline cannot be eliminated completely beprices. cause crude oil is a nonrenewable resource. Therefore, continually higher prices can only reduce the decline rate to a certain degree. However, within a given range of crude oil prices and known existing oil, the decline rate is affected by changes in the price.

In order to have a stable domestic energy policy we must know how changes in the price<sup>14</sup> of oil affect the ultimate supply of oil. How much will a change in price offset

<sup>&</sup>lt;sup>14</sup>As used in this study, "price" does not mean a specific selling price between producer and purchaser and does not represent a future market value. The term "price" is used to refer generally to economic levels which would, on the basis of cases analyzed, support given levels of activity for the particular fuel. Grayson, Leslie F. p. 134.

the decline rate?<sup>15</sup> This depends upon the size of the price change, the elasticity of supply with respect to price, and the size of the decline rate. The effect of the decline rate upon crude oil production causes it to be an essential variable in forecasting oil supply. Given this importance in forecasting, the decline rate again exhibits its importance and its policy implications.

The price of oil gives a starting point at time zero and aids in constructing a supply curve for an individual well. Figure Two shows that price has a definite effect on the quantity of crude oil supplied.<sup>16</sup> The U.S. is depleting its resources of low cost petroleum. This is not to say there are not substantial amounts of oil left, but only that it will be more expensive to extract. Therefore, the appropriate supply question is not "how much petroleum is left?" but "how much petroleum is left at different prices?"<sup>17</sup>

The third variable mentioned is the rate of change in the price of oil. In the past years the U.S, has experienced a rapid increase in the price of oil. Not only does the price level have an effect on the supply of oil, but also the rate at which prices change has an effect. In this study price will increase at a constant rate per period.

<sup>17</sup>Uhler, Russell, p. 6.

<sup>&</sup>lt;sup>15</sup>This will be discussed extensively in Chapter Four.

<sup>&</sup>lt;sup>16</sup> It is assumed here that in order to supply the oil it must first be recovered. This study is not concerned with oil that has already been extracted.



FIGURE TWO: The marginal cost curve for an individual oil well (which is nothing more than a supply curve for an individual oil well).

To say that an increase in price causes an increase in the quantity of oil supplied implies an upward sloping supply curve such as that exhibited in Figure Two. Exactly how much is the change in quantity for a given change in price depends upon the slope of the supply curve, or the elasticity of supply with respect to price. "It is clear that the magnitude of the initial increment which becomes worthwhile to recover depends crucially on the position and shape of the marginal cost curve."<sup>18</sup>

Production from an individual oil well will decrease through time, ceteris paribus. The total costs of production are constant, but output is decreasing. Therefore, the cost per incremental barrel of oil, marginal cost, will rise. Average costs, cost per barrel, also rise. An increasing marginal cost function yields a positively sloped marginal cost curve for an individual oil well. An individual oil well is the price taker, as in a purely competitive market. In a purely competitive market, profit maximizing firms equate marginal cost with price, points A and B in Figure 2. Thus the marginal cost curve for an individual firm will become that firm's short run supply curve. The steepness of the positively sloped supply curve (i.e., the more steeped the more inelastic the supply) for an oil well is determined by the thickness of the oil in the well, the size of the hole drilled, the gas content in the well, the spacing of wells

<sup>18</sup>Ibid., p. 16.

in an oil field, and other geologic and physical boundaries.

The essential variables and their relative importance have been outlined. The next step is to build a mathematical model that will yield the output of crude oil in any time period in the future and, thus, simultaneously yield a decline curve.

#### CHAPTER THREE

### THE DERIVATION OF A PRODUCTION DECLINE MODEL

The first step in building a mathematical model to give the output of oil in any time period is to isolate the decline rate and see how it affects output while holding the other variables constant.

#### The Decline Rate Isolated

In order to derive the output quantity in each successive time period the decline rate must be accounted for. Let "λ" equal the decline rate.

Example: 10 barrels per day output in time t

 $\lambda = .10$ 

 $10 \times .10 = 1$ 

1 = Production decline in number of barrels in subsequent period

10 - 1 = 9

The production in time t + 1 will be one barrel less than in time t, or 9 barrels. In order to get directly to the production in the subsequent time period with only one step it is necessary to subtract  $\lambda$  from 1 and multiply by the previous production amount.

(I)  $Q_{t+1} = (1 - \lambda) Q_{t} = (1 - .10)10 = 9$ (II)  $Q_{t+2} = (1 - \lambda) Q_{t+1}$ Substitute the expression for  $Q_{t+1}$  into formula II. (III)  $Q_{t+2} = (1 - \lambda) (1 - \lambda) Q_{t}$  Now reduce this to get a general formula for production in any time period given a constant price and accounting for the decline rate.

$$(IV) Q_{t+n} = (1 - \lambda)^{n} Q_{t}$$

where n = number of periods.

The decline rate will cause the supply curve to shift to the left as time passes as shown in Figure Three. The information in Figure Three can be used to derive a decline curve such as that shown in Figure Four (constant price).

The second step in building a model to give output in any time period is to isolate the change in price to see how this affects output while holding the other variables constant.

#### The Change In Price Isolated

In order to get the output quantity for different price levels this work will allow for a constant increase in the price per barrel of oil. Let "P" equal price and "X" equal the constant rate of increase in the price.

Example: Price = 30.00 per barrel in time t X = .05

 $30.00 \times .05 = $1.50$ 

\$1.50 is the increase in price in the subsequent time period.



FIGURE THREE:

Production decline over time with no real change in price.



FIGURE FOUR: The crude oil production decline curve given a constant decline rate and no change in price.
30.00 + 1.50 = 31.50

The price in time t + 1 will be \$1.50 more than it was in time t or \$31.50. In order to get directly to the price in the subsequent time period with only one step it is necessary to add X to 1 and multiply by the previous price.

$$(V)$$
 P<sub>++1</sub> = (1 + X) P<sub>+</sub> = (1 + .05) 30.00 = \$31.50

(VI) P<sub>t + 2</sub> = (1 + X) P<sub>t + 1</sub>

Substitute the expression for  $P_t + 1$  into formula VI. (VII)  $P_t + 2 = (1 + X) (1 + X) P_t$ 

Now reduce this to get a general formula for price in any time period.

$$(VIII) P_{t} + n = (1 + X)^{n} P_{t}$$

where n = number of periods Given an upward sloping supply (or marginal cost) curve, increases in the price of oil will cause increases in the output of oil, and vice-versa as shown in Figure Five.

The final step in building the model is to combine the decline rate, the rate of change in the price, and an elasticity of supply with respect to price to determine their combined effect upon crude oil production.



FIGURE FIVE:

Relationship of quantity to changes in the price. This graph illustrates how changes in the price can affect output but, it also shows the importance of the elasticity of supply with respect to price (i.e. slope of the supply curve) and how it governs the magnitude of the change in quantity for a given change in price. For example, at price  $P_t$ , output equals  $q_t$ . Price rises to  $P_{t+1}$  and output rises to  $q'_{t+1}$ , a larger increase, with  $Q'_t$ . Curve  $Q'_t$  is more elastic in supply than  $Q_t$ . That is, for a given price change,  $P_t$  to  $P_{t+1}$ , the change in output for  $Q'_t$  is greater than the change for  $Q_t$ .

#### The Model

The elasticity of supply with respect to price shows how much quantity will change for a given change in price. Earlier the supply curve was denoted as  $Q_t = f(P)$ . It is now necessary to make this more specific and include a value for elasticity. The representation for the supply curve will now be formula IX.

(IX)  $Q_t = AP_t^e$ where: A = a positive constant

P = price

e = elasticity of supply with respect to
 price<sup>1</sup>

Now include into formula IX the general formula for production in any time period accounting only for the decline rate -- formula IV. This formula will yield the supply curve for any time period.

(X)  $Q_{t + n} = A (1 - \lambda)^{n} P_{t}^{e}$ 

Now include into this formula the general formula for price in any time period -- formula VIII.

(XI) 
$$Q_{t+n} = A (1 - \lambda)^{n} [(1 + \chi)^{n} P_{t}]^{e}$$

Now simplify equation XI to get a General Formula that will yield the quantity output for any period in the future given a decline rate, a price level, a rate of change in price,

<sup>&</sup>lt;sup>1</sup>See Appendix I for a mathematical proof that "e" actually equals the elasticity of supply with respect to price.

(XII) 
$$Q_{t+n} = A (1 - \lambda)^{n} (1 + x)^{en} p_{t}^{e}$$

Given this formula and a value for each variable it is possible to determine quantity output for any time period in the future and thereby derive a decline curve as exhibited in Figures Six and Seven.

In order to test this formula for accuracy it is necessary to compute production for each subsequent period based on the prior period's production through the use of the shorter formulas;

$$Q_{t + n} = A (1 - \lambda)^{n} (P_{t + n})^{e}$$

where

 $P_{t + n} = (1 + x)^{n} P_{t}$ 

and then compare these output quantities with those derived by using the General Formula (XII). For any given time period the two methods do in fact yield the same output as is displayed numerically in Appendix II.

A mathematical model, from which to derive a decline curve, has not been built. The next step is to determine the interactions between the variables in this model. Once these interactions have been determined it will be possible to draw conclusions about the importance of these variables in domestic energy policy formation.

8



 $A = q_t$   $B = q_t + 1$   $C = q_t + 2$   $D = q_t + 3$ 

FIGURE SIX: Production changes over time given a decline rate, rate of change in price, and elasticity of supply.

The decline rate is constant. It is a natural physical rate of decline in production at the wellhead. The change in price may offset the decline in production in the short run (as exhibited in Figure Eight) but it doesn't actually change the decline rate and cannot offset the decline indefinitely because oil is a non-renewable resource.



FIGURE SEVEN: The crude oil production decline curve given the quantities in Figure Six.



FIGURE EIGHT: The offset in the decline rate due to an increase in price.

No change in price, quantity produced drops to  $q_t + 1$ . Increase in price to  $P_{t+1}$ , quantity goes to  $q_{t+1}$ . The amount of offset in the decline rate is equal to the difference  $q_{t+1}$  and  $q_{t+1}$ .

# CHAPTER FOUR

# EMPIRICAL ESTIMATIONS AND SIMULATIONS

The model that has been built contains three variables that can affect the future production of oil. These variables are the decline rate, the rate of change in the price of oil, and the elasticity of supply to price. Here the decline rate is a constant percentage reduction in production through time. Because the decline rate is a natural physical phenomenon it is determinable in the real world. The price per barrel of crude oil (p) and the elasticity of supply to price are also determinable. However, the rate of change in price (X) is not as easily determined.

The purpose of this paper is to build a decline curve and therefore determine the output of oil from a well at any time in the future. Given an elasticity of supply to price, a decline rate, and an initial price, how do changes in the price affect the ultimate output of oil from a well? In the past (pre 1981) there has been no change in the price of domestic crude oil because of price controls. This effectively rendered the rate of change in price to zero, X = 0.It is interesting to see how a rate of change in price equal to zero affects output under three different elasticities. It is not necessary to change the decline rate because given no change in price, production will obviously decrease in each successive period using any decline rate. The three levels of supply elasticity used

will be .5, 1, and 1.5 in order to signify all levels of elasticity.<sup>1</sup>

Production of Crude Oil Under Price Controls

A = 2000  $\lambda = .10$ x = .0 P<sub>t</sub> = \$6.00 per barrel n = 0, 1, 2, 3 General Formula: Q<sub>t</sub> + n = A(1 -  $\lambda$ )<sup>n</sup> (1 + x)<sup>en</sup> P<sup>e</sup><sub>t</sub> because x = 0 this equation reduces to<sup>2</sup> Q<sub>t</sub> + n = A (1 -  $\lambda$ )<sup>n</sup> P<sup>e</sup><sub>t</sub> Q<sub>t</sub> + 0 = 2,000 (1 - .10)<sup>0</sup> (6).<sup>5</sup> Q<sub>t</sub> = (2,000) (2.45) Q<sub>t</sub> = 4,900 bls. per time period

By making the same mechanical steps as above the production for each time period can be obtained. These production amounts are as follows:

۷.

 $(1 + x)^{en}$ ; 1 + 0 = 1, one raised to any power equals 1.

<sup>&</sup>lt;sup>1</sup>.5 = relative price inelasticity of supply; 1 = relative unitary price elasticity of supply; 1.5 = relative price elasticity of supply. For further understanding of the concept of elasticity see C.E. Ferguson and J.P. Gould, "Microeconomic Theory" Fourth Edition. Richard D. Erwin, Inc., Homewood, Illinois, 1975.

 $Q_{t} + 1 = 4,410$   $Q_{t} + 2 = 3,969$   $Q_{t} + 3 = 3,572$ e = 1

The other variables remain the same and the calculations are made under the same format.

> $Q_t = 12,000$  bls. per time period  $Q_t + 1 = 10,800$   $Q_t + 2 = 9,720$  $Q_t + 3 = 8,748$

 $Q_t = 29,400$  bls. per time period  $Q_t + 1 = 26,460$   $Q_t + 2 = 23,814$  $Q_t + 3 = 21,433$ 

In each case (e = .5, e = 1, e = 1.5) the decrease in production in each subsequent time period is exactly equal to the decline rate -- 10%. If price controls continued beyond 1981 the U.S. would have experienced a continual decline in output of domestic crude oil equal to the relevant decline rates. In 1981 President Reagan ended all controls on the price of domestic crude oil. This caused an initial increase in production because of higher returns associated with each barrel produced. But what will happen to production in subsequent time periods? This depends upon the relationship between the decline rate  $\lambda$ , the rate of change in the price X, and the elasticity of supply e. Assuming supply has unitary elasticity with respect to price, e = 1, what relationship exists between the change in price (which is a constant percentage rate of change) and the decline rate? In other words, what is the relationship of production in time period t + 1 to the production in time period t, when the elasticity equals one? This relationship is as follows.<sup>3</sup> Given e = 1

 $x = \frac{\lambda}{1 - \lambda} \quad \text{then } Q_{t} + 1 = Q_{t}$  $x > \frac{\lambda}{1 - \lambda} \quad \text{then } Q_{t} + 1 > Q_{t}$  $x < \frac{\lambda}{1 - \lambda} \quad \text{then } Q_{t} + 1 < Q_{t}$ 

When x is equal to  $\frac{\lambda}{1-\lambda}$  the production of oil in time period t + 1 will be exactly equal to the production in time t. This means the change in quantity due to the decline rate is exactly offset by the change in price as

4

<sup>&</sup>lt;sup>3</sup>For the mathematical proof of these relationships see Appendix III.

seen in Figure 9.

When x is greater than  $\frac{\lambda}{1-\lambda}$  The production is greater in time t + 1 than in time t. This means the change in price more than offsets the decline rate as seen in Figure Ten. These first two cases are short run occurrences. They cannot continue indefinitely because oil is a non-renewable resource.

When x is less than  $\frac{\lambda}{1-\lambda}$ , but greater than zero, the change in production due to the decline rate is partially offset by the change in production due to the change in price. However, the quantity in time t + 1 is still less than the quantity in time t. This is exhibited in an earlier section in Figure 8. (Page 12, Chapter 3)

In order to cover all possibilities with this model it is necessary to explore the relationship between x and  $\lambda$ when the elasticity is other than unitary (i.e., inelastic, and elastic). What is the relationship of production in time t + 1 to production in time t given an inelastic marginal cost curve (supply curve)?<sup>4</sup> Given e = .5

$$x = \frac{\lambda(2 - \lambda)}{1 - \lambda^2} \text{ then } Q_t + 1 = Q_t$$
$$x > \frac{\lambda(2 - \lambda)}{1 - \lambda^2} \text{ then } Q_t + 1 > Q_t$$
$$x < \frac{\lambda(2 - \lambda)}{1 - \lambda^2} \text{ then } Q_t + 1 < Q_t$$

<sup>&</sup>lt;sup>4</sup>For mathematical proofs of these relationships see Appendix III.



FIGURE NINE:

Change in quantity due to decline rate is exactly offset by the change in price.  $X = \frac{\lambda}{1 - \lambda}$ 



the second second

FIGURE TEN:

Change in quantity due to decline rate is more than offset by change in price

$$X > \frac{\lambda}{1 - \lambda}$$

When x is equal to  $\frac{\lambda(2-\lambda)}{1-\lambda^2}$  the production of oil in

time period t + 1 will be exactly equal to the production in time t. This means the change in quantity due to the decline rate is exactly offset by the change in quantity due to the change in price. This is the same principle as exhibited in Figure 9 but the magnitude of the changes are different because of an assumed inelastic supply curve (more steep supply curve e = .5).

When x is greater than  $\frac{\lambda(2-\lambda)}{1-\lambda^2}$  then the production of oil in time period t + 1 will be greater than the production in time t. This means the change in quantity due to the decline rate is more than offset by the change in quantity due to the change in price. This is the same principle as exhibited in Figure 10 but, the magnitude of the changes are again different because of an assumed inelastic supply curve (more steep supply curve e = .5). Both of these situations are again only short run phenomena because oil is a nonrenewable resource and therefore exhibits diminishing returns.

When x is less than  $\frac{\lambda(2 - \lambda)}{1 - \lambda^2}$ , but greater than zero, the change in production due to the decline rate is partially offset by the change in production due to the change in price. However, the quantity in time t + 1 is still less than the quantity in time t.

8

The last relationship to be covered is one of a relatively elastic supply curve.

Given e = 2

$$x = \frac{1}{(1 - \lambda)} \cdot 5 - 1 \text{ then } Q_{t} + 1 = Q_{t}$$

$$x > \frac{1}{(1 - \lambda)} \cdot 5 - 1 \text{ then } Q_{t} + 1 > Q_{t}$$

$$x < \frac{1}{(1 - \lambda)} \cdot 5 - 1 \text{ then } Q_{t} + 1 < Q_{t}$$

For each respective situation at e = 2 the same explanations apply that were made for e = 1 and e = .5. The only difference here is that the magnitude of the changes differ because the supply curves are assumed to be relatively elastic (flatter curves, e = 2).

According to the principles of elasticity the less elastic (moving from elastic e >1, to unitary elasticity e = 1, to inelastic e < 1) the supply curve is the greater the change in price will have to be to achieve the same change in output.<sup>5</sup> Given the current model and variable designations this is proven mathematically in Appendix IV.

The only situation not covered thus far is the case of a negative change in the price of crude oil. At the time of this writing it appears the price of oil will indeed decrease

<sup>&</sup>lt;sup>5</sup>See any principles of microeconomics text for a discussion on the principles of elasticity.

somewhat in the near future. What happens to the quantity output of oil in time t + 1 when there is a decrease in price? The answer is easily deduced from the previous calculations. When the rate of change in the price of oil was assumed to equal zero, production decreased by exactly the amount of the decline rate. When any positive value for x was chosen at any level of elasticity there was either an increase in production or a decrease less than the decline rate. From these examples it is easily seen that a decrease in the price of oil would cause a decrease in production greater than the decrease caused by the decline rate.<sup>6</sup>

All possible relationships between  $\lambda$ , x, and e have now been established. The importance of these relationships to this work and their implications concerning domestic energy policy is discussed in the next chapter.

<sup>6</sup>There is an example of this situation in Appendix V.

10

# CHAPTER FIVE

# CONCLUSIONS AND POLICY IMPLICATIONS

This country's foreign policy position is weak on several fronts. These weaknesses are due to energy considerations, mainly crude oil. In order to improve its foreign policy the United States must first improve its domestic energy policy. Because oil is this country's main source of energy (and will continue to be for the next twenty years) accurate forecasts of future supplies of oil are essential to building a timely and effective domestic energy policy.

There are many available methods for forecasting future supplies of oil including geologic, statistical, and economic methods. It has been shown that the most appropriate model for determining the future supply of oil from an individual oil well includes both geologic and economic considerations. Such a model was constructed in this paper and arranged into a general form to give output in any time period in the future. The model is:

 $Q_{t + n} = A (1 - \lambda) (1 + x)^{en} P_{t}^{e}$ 

 $Q_{t + n} = Quantity$  in any time period in the future A = Some positive constant  $\lambda = The production decline rate$  x = The rate of change in the price e = The elasticity of supply with respect to price  $P_t = The initial price$ n = The time period in question From the output projections obtained using this model a decline curve can be derived such as that exhibited in Figure Four (p. 4, Ch. 3). This decline curve shows the output of crude oil for successive time periods in the future given a set of values for the variables in the model.

This model shows that the future quantity output of oil from an individual oil well is a function of the decline rate  $\lambda$ , the rate of change in the price of crude oil x, the base or beginning price of crude oil P (on a per barrel basis), the elasticity of supply with respect to price e, and the time period in question n.

It has been shown that the larger the decline rate the smaller will be the subsequent output of oil. It has also been demonstrated that the larger the rate of increase in the price of oil the higher will be the subsequent output of oil. And, also, it was shown that the more elasticity with respect to price exhibited by the supply curve for an individual oil well, the smaller will need be the increase in the price of oil to offset the decline rate.

Because of the specific relationships between  $\lambda$ , x, and e, that were determined in Chapter Four it is possible to construct various scenarios of future oil supplies based upon different values of these variables. Using these estimates of future supplies of oil, decline curves can be derived to show the production of oil at various time periods

2

in the future.

Example: 
$$e = .5$$
  
 $\lambda = .10$   
 $x = .15$   
 $x < \frac{(2 - \lambda)}{1 - \lambda^2}$ , therefore  $Q_{t + 1}$  is less than  $Q_t$   
.15 < .192

But if e actually turns out to be 2, then

 $x > \frac{1}{(1 - \lambda)} \cdot 5 = 1$ , therefore  $Q_{t+1}$  is greater than  $Q_t$ " .15 > .054

Using these relationships to build various scenarios of the future production of oil, a domestic energy policy can be constructed that relates to the most likely values of the variables in this model. The policy-makers can also have readily available information on the quantity of oil given a change in any of the variables. This builds in a flexibility that is highly needed due to current upheavals in the energy marketplace.

The obvious important variables in this model are  $\lambda$ , x, and e. X and e are subject to change in the marketplace;  $\lambda$  is not. The decline rate is a natural physical phenomenon and can be determined for any individual oil well given that well's specifications. The rate of change in price and supply elasticity are important only in how they relate to the decline rate, as is clearly demonstrated in this paper. This makes the decline rate an extremely important variable in constructing domestic energy policy. Changes in price and supply elasticity are meaningless unless the relevant decline rate is known. The decline rate can be completely offset in the short run but, it is insurmountable in the long run because oil is a non-renewable resource.

Until an alternate energy source as versatile and convenient as oil is perfected it is the responsibility of the U.S. policy-makers to construct a responsible, timely and effective energy policy to protect the energy needs of this country. With knowledge about the relationships between the key variables determining the supply of oil, a beneficial energy policy is more likely. Given such a domestic energy policy the United States' foreign policy situation is greatly improved. This helps to insure maintenance of this country's economic base and industrial strength, thereby protecting the American way of life.

4

#### APPENDIX I

Q<sub>t</sub> = AP<sup>e</sup><sub>t</sub> where "e" equals the elasticity of supply with respect to price.

Purpose: To prove that "e" actually equals the elasticity of supply with respect to price.

Elasticity of supply with respect to price actually equals

<u>% ΔQ</u> % ΔP

It is necessary to rewrite this in calculus terminology

$$\frac{dQ}{Q}$$
  
 $\frac{dP}{P}$ 

and then rearrange the terms to get

 $\frac{dQP}{dPQ} = elasticity of supply with respect to price$ Solve Q<sub>t</sub> = AP<sup>e</sup><sub>t</sub> for "e" to determine if "e" actually equals  $\frac{dQP}{dPQ}$ 

1st: Take the first derivative of both sides with respect to price  $\frac{dQ_t}{dP_+} = e AP_t^{e-1}$ 

2nd: Multiply both sides by  $\frac{P}{Q}$ 

$$\frac{dQ_tP}{dP_tQ} = eAP_t^{e-1} \frac{P}{Q}$$

### APPENDIX I (continued)

3rd: Substitute the original equation for Q  $(Q_t = AP_t^e)$ into the right side of the above equation.

$$\frac{dQ_tP}{dP_tQ} = \frac{eAP_t^{e-1}P}{AP_t^{e}}$$

Since  $P_t^{e-1} \cdot P = P_t^e$  the above equation reduces to

$$\frac{dQ_tP}{dP_tQ} = \frac{eAP_t^e}{AP_t^e}$$

The  $AP_t^e$  cancels out leaving only "e" on the right side.

$$\frac{dQ_tP}{dP_tQ} = e$$

This proves "e" equals the elasticity of supply with respect to price.

### APPENDIX II

### NUMERICAL TEST OF THE MODEL

Definition of all variables:

- Q<sub>t</sub> = Quantity of crude oil production in barrels per time period t.
- n = The time period for which calculating productionA = Some positive constant
- $\lambda$  = The constant decline rate per time period
- x = The constant rate of increase in price per period
   P<sub>t</sub> = The price of crude oil per barrel in time period t
   e = The elasticity of supply with respect to price

Values assigned to variables:

A = 2000 e = .5  $\lambda = .1$   $P_t = $16$  n = 0, 1, 2, 3x = .075

In time period t:

 $Q_t = AP_t^e$   $Q_t = 2000 (16)^{.5}$   $Q_t = 2000 (4)$  $Q_t = 8000 \text{ bls. per time period}$ 

#### APPENDIX II (continued)

In time period t + 1:

 $Q_{t+n} = A (1 - \lambda)^{n} (P_{t+n})^{e}$ Base Formula = where  $P_{t+n} = (1 + x)^{n} P_{+}$  $P_{++1} = (1 + .075)^1 16$  $P_{t+1} = (1.075)16$  $P_{++1} = $17.20$  $Q_{t+1} = 2000 (1 - .10)^{1} (17.20)^{.5}$  $Q_{t+1} = (1800) (4.147)$  $Q_{t+1} = \frac{7465 \text{ bls.}}{1}$  $Q_{t + n} = A(1 - \lambda)^{n} (1 + x)^{en} P_{t}^{e}$ General Formula =  $Q_{t} + 1 = 2000 (1 - .10)^{1} (1 + .075)^{(.5)(1)}$ (16).<sup>5</sup>  $Q_{t+1} = (1800) (1.0368) (4)$  $Q_{t+1} = \frac{7465 \text{ bls.}}{1}$ Base Formula in time t + 1 yields 7465 bls. General Formula in time t + 1 yields 7465 bls. In time period t + 2: Base Formula =  $Q_{t+n} = A (1 - \lambda)^n (P_{t+n})^e$ where

14 . 1 Mar

## APPENDIX II (continued)

$$P_{t + n} = (1 + x)^{n} P_{t}$$

$$P_{t + 2} = (1 + .075)^{2} 16$$

$$P_{t + 2} = \$18.49$$

$$Q_{t + 2} = 2000 (1 - .10)^{2} (18.49)^{.5}$$

$$Q_{t + 2} = (1620) (4.3)$$

General Formula =  $Q_t + n = A(1 - \lambda)^n (1 + x)^{ch} p_t^c$   $Q_t + 2 = 2000 (1 - .10)^2 (1 + .075)^{(.5)(2)}$   $Q_t + 2 = (1620) (1.075) (4)$  $Q_t + 2 = <u>6966 bls.</u>$ 

Base Formula in time period t + 2 yields 6966 bls. General Formula in time period t + 2 yields 6966 bls.

In time period t + 3:

Base Formula =  $Q_{t+n} = A (1 - \lambda)^{n} (P_{t+n})^{e}$ where

$$P_{t} + n^{=}(1 + x)^{n} P_{t}$$
  
 $P_{t} + 3^{=}(1 + .075)^{3} 16$   
 $P_{t} + 3 = $19.86$ 

### APPENDIX II (continued)

 $Q_{t} + 3 = 2000 (1 - .10)^{3} (19.86)^{5}$   $Q_{t} + 3 = (1458) (4.456)$  $Q_{t} + 3 = \underline{6497.5 \text{ bls.}}$ 

General Formula = 
$$Q_t + n = A(1 - \lambda)^n (1 + x)^{en} P_t^e$$
  
 $Q_t + 3 = 2000 (1 - .10)^3 (1 + .075)^{(.5)(3)}$   
 $Q_t = 3 = (1458) (1.1146) (4)$   
 $Q_t = 3 = \underline{6500 \text{ bls.}}$ 

Base Formula in time period t + 3 = 6500 bls.\* General Formula in time period t + 3 = 6500 bls.

Both formulas yielding the same results proves the general formula to be a valid means of projecting output for any given time period.

\*Differences occur only due to rounding.

Relationship between x and λ  $Q_{t + n} = A (1 - \lambda)^{n} (1 + x)^{en} P_{t}^{e}$ at n = 1, solve to find relationship of  $Q_{t + 1}$  to  $Q_t$  as  $\stackrel{\geq}{=} 1$ .  $\frac{Q_{t+1}}{Q_{+}} = \frac{A(1 - \lambda) (1 + x)^{e} P_{t}^{e}}{AP_{t}^{e}}$ the  $AP_t^e$  cancels out on both sides  $\frac{Q_{t+1}}{Q_{+}} = (1 - \lambda) (1 + x)^{e} \stackrel{\geq}{=} 1$ Solve for x to get relationship between x and  $\lambda$  $(1 + x)^{e} = \frac{1}{1-\lambda} \frac{1}{1-\lambda} = (1 - \lambda)^{-1}$ 1 + x =  $(1 - \lambda)^{-1/e}$  $x = (1 - \lambda)^{-1/e} - 1$ 

Where e = 1 = unitary elasticity  $x = (1 - \lambda)^{-1} - 1$   $x = \frac{1}{1 - \lambda} - 1$   $x = \frac{\lambda}{1 - \lambda}$ When  $x = \frac{\lambda}{1 - \lambda}$  then  $\frac{Q_{t} + 1}{Q_{t}} = 1$ 

# APPENDIX III (continued)

When x >  $\frac{\lambda}{1-\lambda}$  then  $\frac{Q_{t+1}}{Q_{t}}$  >1

When x <  $\frac{\lambda}{1-\lambda}$  then  $\frac{Q_{t+1}}{Q_{t}}$  <1

Where e = .5 = relative inelastic

. ...

x	=	(1	-	$\lambda$ ) <sup>-1/e</sup> -1
x	=	(1	-	$\lambda$ ) <sup>-1/.5</sup> -1
x	=	(1	1	$(\lambda)^2 - 1$

$$x = \frac{\lambda(2 - \lambda)}{1 - \lambda^2}$$
  
When  $x = \frac{(2 - \lambda)}{1 - \lambda^2}$  then  $\frac{Q_{t+1}}{Q_t} = 1$   
When  $x > \frac{\lambda(2 - \lambda)}{1 - \lambda^2}$  then  $\frac{Q_{t+1}}{Q_t} > 1$ 

When  $x < \frac{\lambda(2-\lambda)}{1-\lambda^2}$  then  $\frac{Q_{t+1}}{Q_t} < 1$ 

Where e = 2 = relatively elastic

$$x = (1 - \lambda)^{-1/e} -1$$
  

$$x = (1 - \lambda)^{-1/2} -1$$
  

$$x = \frac{1}{(1 - \lambda)} \frac{1}{2} -1$$

# APPENDIX III (continued)

When	$x = \frac{1}{(1 - \lambda)} \frac{1}{2}$	- 1	then	$\frac{Q_{t+1}}{Q_{t}}$	= 1
When	$x > \frac{1}{(1 - \lambda)} 1/2$	- 1	then	$\frac{Q_{t}+1}{Q_{t}}$	>1
When	$x < \frac{1}{(1 - \lambda)} \frac{1}{2}$	- 1	then	$\frac{Q_{t+1}}{Q_{+}}$	<1

.

Relationship between elasticity and output for a change in price

e = .5, to e = 1

$$\frac{\lambda(2 - \lambda)}{1 - \lambda^2} \stackrel{\geq}{\stackrel{\geq}{=}} 1$$

$$\frac{\lambda}{1 - \lambda}$$

 $\frac{2 - \lambda}{1 - \lambda} \stackrel{\geq}{\leq} 1$ 

Obviously this expression is greater than one for any value of  $\lambda$ . Therefore, to get an equal change in quantity the inelastic supply curve (e = .5) will require a greater change in price than the unitary elastic supply curve (e = 1).

e = 2, to e = 1

$$\frac{\frac{1}{(1-\lambda)^{1/2}} - 1}{\frac{\lambda}{1-\lambda}} \stackrel{\geq}{\stackrel{\geq}{\underset{(1-\lambda)^{1/2}}{\stackrel{\sim$$

## APPENDIX IV (continued)

$$\frac{\left[1-(1-\lambda)^{1/2}\right] (1-\lambda)^{1/2}}{\lambda}$$

$$\frac{(1-\lambda)^{1/2}-(1-\lambda)}{\lambda}$$

$$\frac{(1-\lambda)^{1/2}}{\lambda} - \frac{1-\lambda}{\lambda} = 1$$

This expression is obviously less than one for any value of  $\lambda$ . Therefore, to get an equal change in quantity the elastic supply curve (e = 2) will require a smaller change in price than a unitary elastic supply surve (e = 1).

# APPENDIX V

Negative change in price

A	=	2000	
е	=	1	
λ	=	.10	
n	=	0, 1,	2
Ρ	=	\$36.00	)
x	=	06	
t	+	0:	$Q_t = AP_t^e$
			$Q_t = 2000(36)^1$
			$Q_t = \frac{72,000 \text{ bls.}}{2}$ per time period
t	+	1:	$Q_{t+1} = A(1 - )^{n} (1 + x)^{en} P_{t}^{e}$
			$Q_{t+1} = 2000 (.9)^{1} (106)^{(1)} (1) 36^{1}$
			$Q_{t+1} = (1800) (.94) (36)$
			$Q_t + 1 = \frac{60,912 \text{ bls.}}{200000000000000000000000000000000000$
t	+	2:	$Q_{t+2} = 2000 (.9)^2 (.94)^2 (36)$
			$Q_t + 2 = (1620) (.8836) (36)$
			$Q_t + 2 = \frac{51,532 \text{ bls.}}{2}$ per time period

### APPENDIX V (continued)

In time period t + 1 the quantity produced dropped by 15.4% which is greater than 10% decrease caused by the decline rate. In time period t + 2 the quantity produced dropped 15.4% again. This illustrates the additional decrease in production (beyond that caused by the decline rate) caused by a decrease in the price.






## FIGURE TWELVE:

Decline curve from Figure Eleven for decrease in price,

## BIBLIOGRAPHY

4. 4

- Cook, Earl. Forecasting Depletion In Background Readings on Energy Policy. Edited by the Committee on Ways and Means. Washington, D.C.: U.S. Government Printing Office. 1979, p. 96.
- Enhanced Oil Recovery, "An Analysis of the Potential for Enhanced Oil Recovery from Known Fields in the United States -- 1976, p. 216.
- Erickson, Ed; Millsaps, Steven W.; Peters, William L. "The Benefits of Phased Decontrol of the Price of All U.S. Crude Oil Production." A paper prepared for the <u>American Petroleum Institute</u>. March 28, 1979, p. 6.
- Grayson, Leslie E., Editor. <u>Economics of Energy</u> Princeton, N.J., The Darwin Press. 1975, p. 97.
- Jacoby, Neil H. "Multinational Oil" <u>A Study in Industrial</u> <u>Dynamics</u>. New York: McMillan Publishing Co., Inc. 1974, pp. 263, 267.
- May, C.J. "The Production of Oil and Gas" G.D. Hobson, <u>Modern Petroleum Technology</u>. New York: John Wiley and Sons. 1975, p. 145.
- McKie, James W. "United States and Canadian Energy Policy" <u>Oil in the Seventies</u>. The Fraser Institute, 1977, p. 247.
- Tietenberg, Thomas H. <u>Energy Policy and Planning</u>. D.C. Heath and Co., Lexington, Massachusetts. Lexington Books. 1976, p. 21.
- Uhler, Russel. "Economic Concepts of Petroleum Energy Supply" <u>Oil in the Seventies</u>. The Fraser Institute, 1977, p. 20.